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BOOSTER PARAMETERS FOR 1 OR 2-TURN STACKING IN THE ACCUMULATOR

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The introduction of the "Proton Accumulator"[1] between the booster and the main ring allows the booster to cycle more slowly. This results in a pronounced reduction in its rf system requirements. Alternatively, if 2 turn injection into the accumulator is considered, the charge required per booster cycle is halved, and since the booster injection energy is determined mainly by space charge considerations, this suggests a reduction in the booster injection energy.

The optimization of the booster injection energy with single turn injection into the accumulator ring has been separately considered.[2] This note is mainly concerned with the implications for the booster parameters if 2-turn stacking into the accumulator was to be considered the normal mode of operation.

Criteria for Parameter Scaling

Since it will be accepted that the main ring aperture should not be increased, the transverse phase space areas of the 10 BeV main ring injected beam should not be larger than the present design book values. Directly related to this is the linac beam emittance, its dilution in booster, beam transfer systems and proton accumulator, and its accepted magnitude if different values of the required linac beam intensity are considered.

Here it will be assumed that the linac beam emittance value which will be used as the relevant design value, is proportional to linac beam intensity, at least in the intensity domain of ≅ 50 ma to 100 ma. This does not imply that a "knob" exist which, with reducing linac intensity, will reduce the linac emittance. It does imply that for various maximum currents, different ion source geometrical parameters will be used, permitting the use, as a design parameter, of a lower emittance value for lower linac beam currents, i.e.,

$$A_{2,L} = \pi E_{2,L} = C_1 I_L$$

in domain $50 < I_L < 100$ ma for fixed linac energy.

Further, the following assumptions have been made as related to phase space dilution factors:

a) For the vertical phase space dilution at or near injection energy in the booster a factor of 2 has been assumed. (This has been accepted for the existing booster design). This factor has been assumed to be a function of $(N_{sp.ch.}/N_{st})_B$, i.e., Booster space charge limit divided by stacked number of protons. Arbitrarily, the values of 2, 1.7, 1.5 have been used in the following table depending on the value of $(N_{sp.ch}/N_{st})_B$

b) Four turn Booster injection stacking, with a stacking dilution of $\sigma_{st} = 1.5$ has been used in the calculations, although the "lossy multiturn" injection technique will be used operationally.

c) Transverse phase space dilution factors of 1.1 (10% dilution) due to transport and injection errors, $(1.1)^2$ with the proton accumulator and a value of $\sigma_{st} = 1.3$ for the proton accumulator two-turn injection process ("Coherent" two turn stacking) have been assumed. These values should be considered optimum values, which seem however obtainable.

In the cases to be indicated below, whereby two turn injection into the proton accumulator has been assumed, the lower charge per booster cycle requirement resulted in the possibility of either using one or two turn injection into the booster. All resulting parameters for both options have been calculated, only the two turn injection cases will be given here. The one turn injection into the booster, taking into account $E_{2,L} = CI_L$, resulted in all cases in a significantly higher booster cost and therefore will not be presented here.

In the table below the designations, such as $B_{150,10}S_2$ have been used, referring to the main ring injector variants, whereby a booster with 150 MeV injection energy, cycling frequency of 10 Hz and proton accumulator with two turn injection has been assumed.

The following parameters have been used in all cases:

a) $\dot{N}_{MR}$/sec = 1.5 10^{13}.

b) $\left(\frac{\Delta p}{p}\right)_{\text{linac}}$ at 200 MeV, for 10 < I_L < 100 ma, equals ±0.8 10^{-3}

For various booster injection energies this is scaled with $\frac{\Delta p}{p} \propto \beta^{-1}(\beta\gamma)^{-1/4}$

c) $A_{2,L}$ = π cm-mrad, at I_L = 75 ma.

Table I

Main Ring Injector Variants Comparison

	$B_{200,15}$	$B_{200,5S_1}$	$B_{200,10S_2}$	$B_{150,10S_2}$	$B_{100,10S_2}$
MR rep. rate (sec)	3	2.6	2.6	2.6	2.6
N_{MR}/cycle	4.5 10^{13}	3.9 10^{13}	3.9 10^{13}	3.9 10^{13}	3.9 10^{13}
N_B/ cycle	3.5 10^{12}	3.0 10^{12}	1.5 10^{12}	1.5 10^{12}	1.5 10^{12}
B., ma-turns, inj.	200	175	86.7	78	66
Nominal ma-turns, inj.	4 X 67.5	4 X 60	2 X 60	2 X 55	2 X 45
$A_{2,L}$ (cm-mrad)	0.9π	0.8π	0.8π	0.88π	0.87π
A_{2V} X A_{2H} (at p_i, B) (μrad m)^2	18π/54π	16π/48π	12π/24π	15π/26π	17π/26π
$(N_{spch}/N_{st})_B$	1.4	1.4	2.4	1.8	1.1
A_{2V} X A_{2H} (at p_i, MR) (μrad.m)^2	1.2π/3.5π	1.0π/3.4π	0.8π/4.4π	0.9π/4.1π	0.8π/3.3π

A simple graph (Fig. 1) illustrating these results suggest a favourable optimum, if two turn PA injection is assumed, as $B_{120,10}S_2$. In this case approximately the same $(N_{sp.ch.}/N_{st.})_B$ would be obtained, whereas the horizontal transverse phase space ("grunck" $\equiv$ G) at injection in the main ring would be approximately the same as in the $B_{200,15}$ case.

Surprising as these results may be, it is directly related to the assumption of

$$\varepsilon_{2,L} = C' I_L ,$$

where $\varepsilon_{2,L}$ is the momentum normalized emittance $\left(\varepsilon_2 = \beta\gamma \frac{A_{2,L}}{\pi}\right)$. This may be shown in the following manner by writing further:

$$N_{st,B} = C'' I_L \left(\frac{2\pi R}{ec}\right) \frac{n_B}{\beta_{i,B}} = C_o I_L \frac{n_B}{\beta_{i,B}} \quad .$$

Combining these two equations yields

$$\varepsilon_2 = C \;\beta_{i,B} \cdot N_{st.B}/n_B \; .$$

The horizontal transverse phase space emittance (G_H) available at injection to the main ring is given by

$$G_H = n_{PA} \; d_H \; n_B \; \varepsilon_2$$

where n_{PA} is the number of injected turns into the PA; n_B the same for the booster and the dilution factor, d_H, is given by

$$d_H = d_{st.B} \cdot d_{st.PA} \cdot (d_{tr})^2$$

Consequently, $G_H = C\, n_{PA}\, d_H\, \beta_{i,B}\, N_{st.B}$.

For fixed values of n_{PA}, d_H and $N_{st.B}$, G_H is proportional to $\beta_{i,B}$. This is in general agreement with the results shown in Fig. 1.

Similarly, the expression for G_v can be given as

$$G_v = C\, d_v \cdot \beta_{i,B} \frac{N_{st.B}}{n_B} ,$$

where, in this case the vertical dilution factor may be expressed as

$$d_v = d_{v,B} \cdot d_{v,PA},$$

and

$$d_{v,B} = d_{v,B}\left(\frac{N_{sp.ch.}}{N_{st}}\right).$$

For fixed values of $N_{st,B}$, injected number of turns into the booster n_B, G_v is proportional to $\beta_{i,B}$, however, for various values of $\left(\frac{N_{sp.ch.}}{N_{st}}\right)_B$, a variation of $d_{v,B}$ should be assumed. As indicated in the foregoing, the values of 1.5, 1.7, 2.0 have been assumed in the cases for 200, 150, 100 MeV injection, respectively. This explains the non variation of the

G_v values in the table with booster injection energy, i.e., $\beta_{i,B} \cdot d_v$ for these cases is 0.85, 0.86, 0.85[5]; respectively. This is coincidental related to the somewhat arbitrary choice of the magnitude of $d_{v,B}$.

It is obvious from the expressions for G_H and G_V that a lower booster injection energy is desirable, at least for the two turn injection case. The extent to which it is possible to lower the injection energy can be found, in a first approximation,* as follows:

$$N_{sp.ch.B} = K'\beta^2\gamma^3 A_{2,L} \left(1 + \sqrt{n_B d_{st.B}}\right)$$
$$= K''\beta\gamma^2 \varepsilon_{2,L} \left(1 + \sqrt{n_B \cdot d_{st.B}}\right).$$

Consequently,

$$\left(\frac{N_{sp.ch}}{N_{st}}\right)_B = K(\beta\gamma)^2 \left[\frac{1 + (n_B d_{st.B})^{1/2}}{n_B}\right].$$

This equation suggests that for a fixed mode of booster injection, say nominally $n_B = 4$, $d_{st.B} = 1.5$, as is assumed for the $B_{200,15}$ case, any lowering of $(\beta\gamma)$ results in a lower value of $\left(N_{sp.ch.}/N_{st}\right)_B$, which would be undesirable. However, with two turn injection into the PA the booster charge per cycle has been reduced sufficiently, so that two turn booster injection stacking, with nominally, $n_B = 2$, $d_{st,B} = 1.3$, would be a more favourable mode of injection.

*The variation of the space charge formula image effect factor has been ignored here.

The form in square brackets increases accordingly by a factor of ≅ 1.5, permitting a lower $(\beta\gamma)_{B,inj.}$ value by a factor of $\sqrt{1.5}$, or a booster injection energy of approximately 130 MeV. This is consistent with the conclusions to be drawn from the values in Table I and the results illustrated in Fig. 1.

Conclusions

From the foregoing the following conclusions may be drawn:

1) $B_{E,5}S_1$ cases.

Lower booster injection energy is possible. However, more phase space dilution is required at booster injection in prder to meet the space charge limit. As a consequence a net increase occurs for the G_V and G_H values resulting in reduced beam "brightness." The conclusion is that with single turn PA injection, 200 MeV injection energy into the booster seems to be a desirable parameter when considerin the various factors involved.

2) $B_{E,10}S_2$ cases.

Related to the significantly lower charge per booster cycle, lower injection energy into the booster is possible, as indicated above, and actually essential in order to obtain smaller G_H values for injection into the main ring. With approximately 120 MeV booster injection energy, it seems possible to obtain comparable G_H, G_V values as for the case in which no PA is used.

3) A comparison of the two turn PA injection and single turn PA injection leads to

$$\frac{(G_H)_{1 \text{ turn PA}}}{(G_H)_{2 \text{ turn PA}}} = \frac{1}{d_{st.PA}} \cong 0.77$$

A priori, a factor > 2 would be expected here, the significantly smaller factor traces back again to $\varepsilon_{2,L} = C'I_L$. This is consistent with the G_H values given in column 2 and 3 of Table I. Consequently, for both single and two turn (multiturn) PA injection the lowest injection energy consistent with the booster space charge limit is indicated from the point of view of magnitude of G_H, G_V values.

4) Considering Main Ring injector variants, it is relevant, specifically related to future use of intersecting storage rings, to preserve beam brightness [$\propto N_{MR}/(G_H G_V)$] as much as possible.

5) A sufficient amount of uncertainty related to the transverse phase space dilution factors, especially related to $d_{st.PA}$ exists, so that the actual values assumed here should be considered optimistic values.

6) The foregoing is meant to consider relevant parameters for the booster in case of utilization of a PA. It should not be interpreted as expressing an opinion related to the PA addition as a desirable design variant of the total accelerator.

7) The PA presents a desirable future option for further increasing the main ring average intensity.

ACKNOWLEDGMENT

Additional cost factors presented in the appendix were provided by Q. Kerns and D. Young. Further acknowledgment is made of stimulating discussions with A. Maschke.

REFERENCES

[1]R. R. Wilson, "Injection Accumulator," National Accelerator Laboratory report NAL-11.

[2]R. Billinge, Q. Kerns, L. Teng, G. Tool, A. van Steenbergen, and D. Young, "Injection Energy of the 5 Hz Booster," National Accelerator Laboratory report FN-147.

APPENDIX

For the various cases cost estimates have been made and cost differences have been calculated for Booster Magnet system and power supply, Booster rf system, Linac costs, etc.

This may be summarized as follows (Table II and III):

Table II

Booster PS + Magnet	$B_{200,15}$	$B_{200,5}S_1$	$B_{200,10}S_2$	$B_{150,10}S_2$	$B_{100,10}S_2$
$A_{2V} \times A_{2H}$ (at p_i,B) $(\mu rad.m)^2$	$18\pi/54\pi$	$16\pi/48\pi$	$12\pi/24\pi$	$15\pi/26\pi$	$17\pi/26\pi$
Apertures, $(mm)^2$ (G/2 X H/2) D	28.4 X 47.8	27.2 X 46.2	24.9 X 39.0	27.0 X 40.4	28.3 X 41.
F	21.5 X 68.1	20.6 X 65.5	18.9 X 54.3	20.4 X 56.6	21.4 X 58.
Magnet Stored Energy, MJ	1.0	0.90	0.73	0.84	0.91
Cost PS + Magnet, M$	3.33	3.06	2.44	2.80	3.05

Table III

Δ Cost (M$)	$B_{200,15}$	$B_{200,5}S_1$	$B_{200,10}S_2$	$B_{150,10}S_2$	$B_{100,10}S_2$
Booster PS + Magnet	0	-0.3	-1.0	-0.65	-0.4
Booster rf	0	-1.9	-1.0	-0.7	+0.3
Linac	0	0	0	-1.8	-3.8
MR rep rate	0	-0.3	-0.3	-0.3	-0.3
P A	0	+	+	+	+

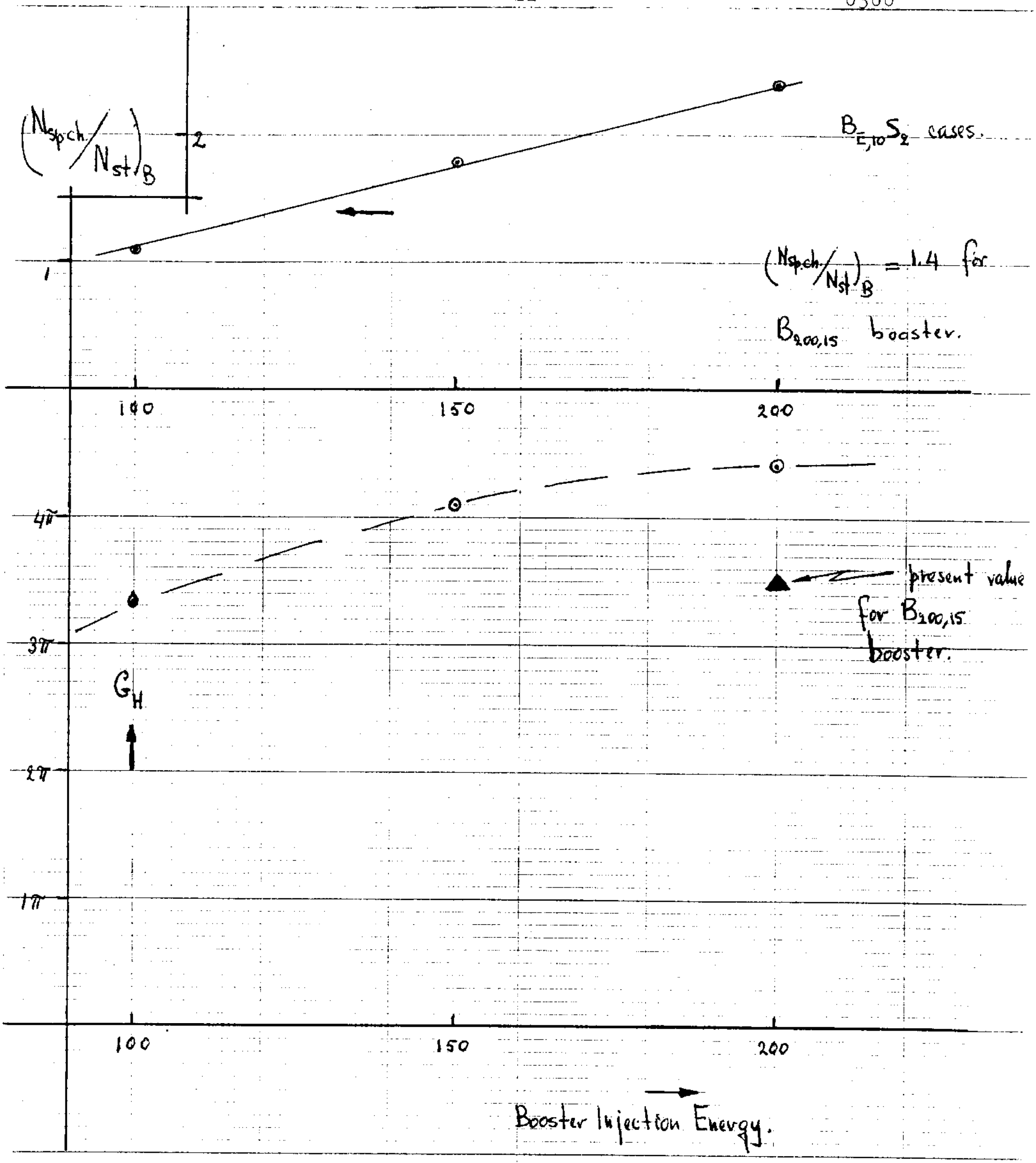
$\left(N_{sp.ch.}/N_{st}\right)_B$
2
1
$B_{E,10}S_2$ cases.
$\left(N_{sp.ch.}/N_{st}\right)_B = 1.4$ for
$B_{200,15}$ booster.
100
150
200
4π
3π
2π
1π
G_H
present value
for $B_{200,15}$
booster.
100
150
200
Booster Injection Energy.

Fig. 1